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Letters to the Editor

On the power spectrum of speckles in coherent imaging

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The speckle pattern in the image is generally reduced by moving the diffuser rapidly. This is, however, not possible in all cases viz. in holographic imagery. To minimise the speckle in holographic imagery a method (Dainty & Welford 1971) which is basically a spatial frequency technique was suggested to have a moving aperture in the pupil plane of the imaging lens. Recently Hariharan & Hegedus (1974) have obtained an expression for the power spectrum of speckles in the image of a coherently illuminated diffusing object in terms of the shift of the sampling aperture in the spatial frequency plane between successive apertures by extending the same procedure to the case of a moving random mask. They have assumed the diffuser to be stationary. In the present communication we would like to show the effect of a moving diffuser associated with a stationary diffuser on the power spectrum of speckles.

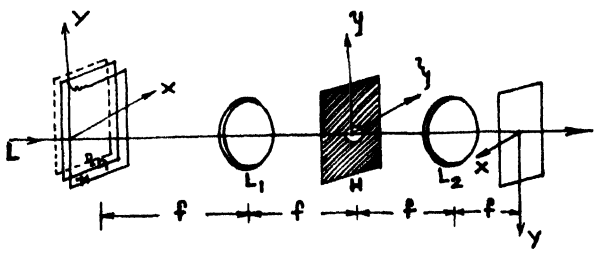

Fig. 1. D_1, D_2 are diffuser, D_1 is stationary, D_2 is mlving. L_1, L_2 lenses. H—Aperture.

Let us consider the coherent optical system as shown in the figure the symbols and notation used therein being the same as those used by Hariharan & Hegedus (1974) for convenience. The system gives the image of an object plane consisting of two diffusers with complex amplitude transmittances $d_1(r)$ and $d_2(r)$ respectively where r is a point specified by the co-ordinates (x, y). Let us assume that the diffuser with transmittance $d_2(r)$ is moving while the other is stationary. The emergent light from the diffusers is modulated by a factor $t(r)$ and the object function can be written as

$$f(r, vt) = t(r)d_1(r)d_2(r-vt) \quad \dots (1)$$

where v is the constant velocity of the moving diffuser. If $h(r)$ is the coherent system impulse then under the assumption of unit magnification the image amplitude can be written as

$$g(r, vt) = f(r, vt) * h(r) \quad \dots (2)$$

where the symbol * denotes convolution.

Now for n equal exposures with a shift of the aperture in between two successive ones the resultant illuminance $I(r)$ at any point in the image plane will be the sum of the individual illuminances and can be written as

$$I(r) = \sum_{i=1}^{n} I_i(r, vt) \quad \dots (3)$$

where $I_i(r, vt) = |g_i(r, vt)|^2$ and $g_i(r, vt) = f(r, vt)*h_i(r)$, $h_i(r)$ being the system impulse for the i-th exposure. The auto-correlation of the resultant illuminance can be written as

$$R_{II}(r_1, r_2, vt) = \sum_{i,j} \langle I_i(r_1, vt)\rangle \langle I_j(r_2, vt)\rangle + \sum_{i,j} |R_{g_i} R_{g_j}(r_1, r_2, vt)|^2 \quad \dots (4)$$

The average power spectrum of the image illuminance can be expressed as the sum of two terms as follows.

$$S_{II}(u_1, u_2, vt) = \Omega_1(u_1, u_2, vt)+\Omega_2(u_1, u_2, vt) \quad \dots (5)$$

where $S_{II}(u_o, u_2, vt)$ is the double Fourier transform of the illuminance $R_{II}(r_1, r_2, vt)$. In other words as we can write Ω_1 and Ω_2 as the respective Fourier transforms of the two terms of the expression (4). It is shown by Lowenthal & Joyeux (1971) that the term $\langle I(r, vt)\rangle$ in the image does not depend either on position r or on time t. The average power spectrum, replacing u_1 and u_2, is given (Hariharan & Hegedus 1974) as

$$\Omega_1(u, u, vt) = |\mathcal{J}(u)\mathcal{H}(u)|^2 \quad \dots (6)$$

where $\mathcal{J}(u)$ is the Fourier transform of the irradiance of the object and $\mathcal{H}(u)$ is the incoherent transfer function of the system. The above equation therefore gives the power spectrum of a spatially incoherent object having the same irradiance $|t(r)|^2$ as the object transparency.

The second term $\Omega_2(u_1, u_2, vt)$ is more important for us as it gives the power spectrum of the speckle. For two linear systems with inputs $f_i(r, vt)$, $f_j(r, vt)$, impulse responses $h_i(r)$, $h_j(r)$ and outputs $g_i(r, vt)$, $g_j(r, vt)$ respectively, the cross-correlation of the outputs can be expressed in terms of the cross correlation of the inputs as (Papoulis 1965).

$$R_{g_i g_j}(r_1, r_2, vt) = R_{f_i f_j}(r_1, r_2, t) *_{r_2} h_j(r_2) *_{r_1} h_i(r/1) \quad \dots \quad (6)$$

where the subscript r is the variable on which the operation is carried out, keeping the other variables constant. Thus

$$\Omega_2(u_1, u_2, vt) \longleftrightarrow \sum_{i,j} | R_{ff}(r_1, r_2, vt) *_{r_1} h_i(r_1) *_{r_2} h(r_2) |^2 \quad \dots \quad (7)$$

where $R\ (r_1, r_2, vt)$ is the auto correlation function of the object amplitude $f(r, vt)$.

If the double Fourier transform of R_{ff} is

$$\Gamma(u_1, u_2, vt) \longleftrightarrow R_{ff}(r_1, r_2, vt) \quad \dots \quad (8)$$

then

$$\Omega_2(r_1, r_2, vt) = \sum_{i,j} | \Gamma(u_1, u_2, vt) H_i(u_1) H_j^*(-u_2) |$$
$$* [\Gamma(u_1, u_2, vt) H_i^*(-u_1) H_j(u_2)]. \quad \dots \quad (9)$$

In order to obtain the average power spectrum of the speckle we have to replace u_1 and u_2 by u. Then $\Gamma(u, u, vt)$ becomes $\bar{S}_{ff}(u, vt)$ and hence

$$\Gamma(u, u, vt) = \bar{S}_{ff}(u, vt) = R_{ff}(0, vt). \quad \dots \quad (10)$$

Hence the average power spectrum of the speckle is given by

$$\Omega_2(u, u, vt) = | R_{ff}(0, vt) |^2 \sum_{i,j} | H_i(u) H_j^*(u) | * [H_i^*(-u) H_j(-u)] \quad \dots \quad (11)$$

The above expression clearly shows that the power spectrum of speckle depends on the moving diffuser. In the absence of the moving diffuser $R_{ff}(0, vt)$ becomes $R_{ff}(0)$ which is constant and the expression (11) is then identical to that derived by Hariharan & Hegedus (1974).

The illustrate the above result, let us consider a rectangular aperture of unit width

$$\begin{aligned} \mathrm{rect}(u) &= 1 \quad |u| < 1/2 \\ &= 0 \quad |u| > 1/2 \end{aligned} \quad \dots \quad (12)$$

which is moving in the Fourier plane with unit velocity along a line parallel to its width during the exposure time T, so that the total distance U through which the aperture moves is $T = U$ which is assumed to be greater than its width.

The displacements of the aperture at any two instants t_1 and t_2 during exposure are given by

$$H_{t_1}(u) = \mathrm{rect}(u-u_1)$$
$$H_{t_2}(u) = \mathrm{rect}(u-u_2) \qquad \dots \quad (13)$$

u_1, u_2 being the initial positions of the aperture. Now proceeding on the same lines as Hariharan & Hegedus (1974) we get the contribution to the power spectrum of the speckle along the direction of the movement from the aperture at those two positions as

$$\Delta\Omega_2(u, u, vt) = |R_{ff}(0, vt)|^2[\mathrm{rect}(u-u_1)\mathrm{rect}(u-u_2)] * [\mathrm{rect}(u_1-u)\mathrm{rect}(u_2-u)|. \qquad \dots \quad (14)$$

Integrating over all values of u_1 and u_2,

$$\Omega_2(u, u, vt) = (1/U^2)|R_{ff}(0, vt)|^2 \int_u^U \int_u^U (1-|u_1-u_2|-u)du_1du_2 \quad \dots \quad (15)$$

where u_1, u_2 and u are always positive and $u_1-u_2 \leqslant 1-u$.

From eq. (15) it is clear that the power spectrum of the speckle is always weighted by the moving diffuser and the same logic can be extended to the case of moving random mask.

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References

Dainty J. C. & Welford W. T. 1971 *Opt. Commun.*, **3**, **289**.
Hariharan P. & Hegedus Z. S. 1974 *Opt. Act.* **21**, **683**.
Lowenthal S. & Joyeux D. 1971 *J. Opt. Soc. Am.* **61**, **847**.
Hariharan P. & Hegedus Z. S. 1974 *Opt. Act.* **21**, **345**.
Papoulis A. 1965 *Probability, random variables and Stochastic Processes*, McGraw-Hill, New York.